5 Introduction to Euclid's Geometry

Fastrack Revision

- **Euclid's Definitions:**
  1. A point is that which has no part.
  2. A line is breadthless length.
  3. The ends of a line are points.
  4. A straight line is a line which lies evenly with the points on itself.
  5. A surface is that which has length and breadth only.
  6. The edges of a surface are lines.
  7. A plane surface is a surface which lies evenly with the straight lines on itself.
- **Statement:** A sentence which can be judged to be true or false, *e.g.*, The sum of the angles of a quadrilateral is 360°, is a true statement and a line segment has one end point, is a false statement.
- **Axioms:** The basic facts taken for granted without proof. *e.g.*, A line has infinitely many points.
- **Theorem:** A mathematical statement whose truth has been established (proved).
- **Euclid's Axioms:**
  1. Things which are equal to the same thing are equal to one another.
  2. If equals are added to equals, the wholes are equal.
  3. If equals are subtracted from equals, the remainders are equal.
  4. Things which coincide with one another, are equal to one another.
  5. The whole is greater than the part.
  6. Things which are double of the same things, are equal to one another.
  7. Things which are halves of the same things, are equal to one another.
- **Postulate:** The assumptions which are specific to geometry, *e.g.*, Two points make a line.
- **Euclid's Postulates:**

  Postulate 1: A straight line may be drawn from any one point to any other point.

  Postulate 2: A terminated line can be produced indefinitely.

  Postulate 3: A circle can be drawn with any centre and any radius.

  Postulate 4: All right angles are equal to one another.

  Postulate 5 (Parallel Postulate): If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

Knowledge BOOSTER

1. *An axiom generally is true for any field in science, while a postulate can be specific on a particular field.*
2. *It is impossible to prove from other axioms, while postulates are provable to axioms.*
3. *Though Euclid defined a point, a line and a plane, the definitions are not accepted by mathematicians. Therefore, these terms are now taken as undefined.*
4. *A system of axioms is called consistent, if it is impossible to deduce a statement from these axioms that contradicts any axiom or previously proved statement.*

Practice Exercise

Multiple Choice Questions

Q 1. The number of dimensions, a solid has:

a. 0 b. 1 c. 2 d. 3

Q 2. The total number of propositions in the elements are:

a. 13 b. 55 c. 460 d. 465

Q 3. 'Lines are parallel if they do not intersect' is stated in the form of:

a. a definition b. an axiom
c. a proof d. a postulate

Q 4. How many lines passes through two distinct points?

a. 1 b. 2
c. 3 d. 4

Q 5. **The number of dimensions a surface has:**
a. 0 b. 1 c. 2 d. 3

Q 6. **Which of the following needs a proof?**
a. Axiom b. Theorem
c. Definition d. Postulate

Q 7. **It is known that, if $a + b = 4$, then $a + b + c = 4 + c$. Then, Euclid's axiom that illustrates this statement is:**
a. I axiom b. II axiom
c. III axiom d. IV axiom

Q 8. **Which of these statements do not satisfy Euclid's axiom?**
a. If equals are added to equals, the wholes are equal.
b. If equals are subtracted from equals, the remainders are equal.
c. Things which are equal to the same thing, are equal to one another.
d. The whole is less than the part.

Q 9. **The things which coincide with one another, are:**
a. equal b. unequal
c. half of same thing d. triple of one another

Q 10. **Euclid stated that all right angles are equal to each other in the form of:**
a. a proof b. an axiom
c. a definition d. a postulate

Q 11. **The things which are double of same thing, are:**
a. equal to one another
b. unequal
c. halves of same thing
d. double of the same thing

Q 12. **The number of lines that can passes through a given point is:**
a. infinitely many b. only two
c. only one d. None of these

Q 13. **Euclid stated that 'things which are equal to the same thing' are equal to one another in the form of:**
a. a definition b. a postulate
c. an axiom d. a proof

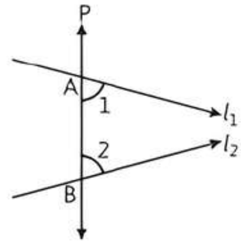
Q 14. **Which of the following statement is incorrect?**
a. A line segment has definite length.
b. Three lines are concurrent, if and only if they have a common point.
c. Two lines drawn in a plane always intersect at a point.
d. A line has no defenite length.

Q 15. **Identify the incorrect statement:**
a. Only one line can pass through a single point.
b. Only one line can pass through two distinct points.
c. A terminated line can be produced indefinitely on both the sides.
d. If two circles are equal, then their radii are equal.

Q 16. **If the point P lies in between M and N, and C is the mid-point of MP, then:**
a. MC + CN = MN b. MC + PN = MN
c. MP + CP = MN d. CP + CN = MN

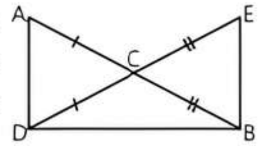
Q 17. **Sohan is of the same age as Mohan. Ram is also of the same age as Mohan. State the Euclid's axiom that illustrates the relative ages of Sohan and Ram.**
a. First Axiom b. Second Axiom
c. Third Axiom d. Fourth Axiom

Q 18. **If the given figure, if $\angle 1 + \angle 2 < 180°$, then l_1 and l_2 will eventually meet at:**



a. left side of AB
b. right side of AB
c. either side of AB
d. will never meet

Q 19. **In the adjoining figure, AC = DC and CB = CE. Using an Euclid's axiom, we have:**



a. AB ≅ DE b. AB ≅ 2DE
c. AD = BE d. None of these

Assertion & Reason Type Questions

Directions (Q. Nos. 20-24): *In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct option:*
a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
c. Assertion (A) is true but Reason (R) is false.
d. Assertion (A) is false but Reason (R) is true.

Q 20. **Assertion (A): There can be infinite number of lines that can be drawn through a single point.**
Reason (R): From a single point, we can draw only two lines.

Q 21. **Assertion (A): According to the Euclid's first axiom, 'Things which are equal to the same thing are also equal to one another'.**
Reason (R): If AB = MN and MN = PQ, then AB = PQ.

Q 22. **Assertion (A): According to Euclid's second axiom, when equals are added to equals, then wholes are equal.**
Reason (R): Anil and Mukesh have the same weight. If they each gain weight by 3 kg, second Euclid's axiom will be used to compare their weights.

Q 23. Assertion (A): If lines AB, AC, AD and AE are parallel to line l, then the points A, B, C, D and E are collinear.

Reason (R): Infinite lines can be drawn through A and Parallel to l.

Q 24. Assertion (A): Euclid fifth postulate imply the existence of parallel lines.

Reason (R): If the sum of the interior angles will be equal to sum of the two right angles, then two lines will not meet each other on either sides and therefore they will be parallel to each other.

Fill in the Blanks Type Questions

Q 25. A point has .... dimension.

Q 26. The shape of base of pyramid is any ....

Q 27. There are .... number of Euclid's postulates.

Q 28. Euclid stated that all right angles are equal to each other in the form of a .... (axiom/postulate).

Q 29. Two distinct points in a plane determine a .... line.

True/False Type Questions

Q 30. Things which are halves of the same thing, are equal to one another.

Q 31. In postulate 2, a circle can be drawn with any centre and any radius.

Q 32. A terminated line can be produced indefinitely on both the sides.

Q 33. Only one line can pass through a single point.

Q 34. If two circles are equal, then their radii are not equal.

Solutions

1. (d) A solid has three dimensions.
2. (d) The total number of propositions in the elements are 465.
3. (a) a definition
4. (a) Only one line pass through two distinct points.
5. (c) A surface has two dimensions.
6. (b) Theorem needs a proof.
7. (b) We have $a + b = 4$

TR!CK

Using axiom II, if equals are added to equals, the wholes are equal.

$\therefore \quad a + b + c = 4 + c.$

8. (d) 'The whole is less than the part' do not satisfy Euclid's axiom.
9. (a) The things which coincide with one another are equal.
10. (d) Euclid stated that all right angles are equal to each other in the form of a postulate.
11. (a) The things which are double of same thing, are equal to one another.
12. (a) The number of lines that can pass through a given point is infinitely many.
13. (c) Euclid stated that 'things which are equal to the same thing' are equal to one another in the form of an axiom.
14. (c) The incorrect statement is 'Two lines drawn in a plane always intersect at a point'.
15. (a) The incorrect statement is Only one line can pass through a single point'.
16. (a) MC + CN = MN
17. (a) Euclid's First axiom state that illustrates the relative ages of Sohan and Ram.
18. (b) By Euclid postulates if $\angle 1 + \angle 2 < 180°$, then the lines will eventually intersect at the point on the right side of AB.
19. (a) We have, AC = DC and CB = CE

TR!CK

Using Euclid's axiom 2, if equals are added to equals, then wholes are equal.

$\therefore \quad AC + CB = DC + CE$

$\Rightarrow \quad AB = DE$

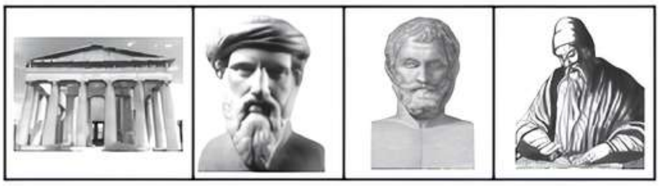
20. (c) Here Assertion (A) is true but Reason (R) is false.
21. (a) Here both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
22. (a) Here both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
23. (c) Assertion (A): It is true to say that, if lines AB, AC, AD and AE are parallel to line l, then the points A, B, C, D and E are collinear.
Reason (R): It is false to say that infinite lines can be drawn through A and parallel to l.
Hence, Assertion (A) is true but Reason (R) is false.
24. (a) Here both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
25. A point has zero dimension. i.e., it has dimensionless.
26. The shape of base of pyramid is any polygon.
27. There are five number of Euclid's postulates.
28. Euclid stated that all right angles are equal to each other in the form of a postulate.

29. Unique
30. True
31. False, because given statement is of postulate 3.
32. True
33. True
34. False

Case Study Based Questions

Case Study 1

A National Public School organised an education trip to a museum. Almost all the students of class IX went to the trip with their teacher of Mathematics. They saw many pictures of mathematicians and read about their contributions in the field of Mathematics. After visiting the museum, teacher asked the following questions from the students.



On the basis of the above information, solve the following questions:

Q 1. Pythagoras was a student of:

a. Euclid b. Thales
c. Archimedes d. Both a. and b.

Q 2. Name of the mathematician who is visible in the last picture, is:

a. Euclid b. Pythagoras
c. Thales d. None of these

Q 3. Euclid stated that 'A circle can be drawn with any centre and any radius', is a/an:

a. definition b. postulate
c. axiom d. proof

Q 4. In which country Thales belong to?

a. Greece b. Egypt
c. Babylonia d. Rome

Q 5. Which of the following needs a proof?

a. Definition b. Theorem
c. Axiom d. Postulate

Solutions

1. (b) Pythagoras was a student of Thales.
So, option (b) is correct.
2. (a) Euclid mathematician is visible in the last picture.
So, option (a) is correct.
3. (b) Euclid stated that 'A circle can be drawn with any centre and any radius' is postulate.
So, option (b) is correct.
4. (a) Thales belongs to Greece Country.
So, option (a) is correct.
5. (b) Theorem needs a proof.
So, option (b) is correct.

Case Study 2

In a class of Mathematics, the teacher taught a chapter 'Introduction to Euclid's Geometry' in which they taught about different postulates and axioms.

On the basis of the above information, solve the following questions:

Q 1. How many axiom's are exist in Euclid's?

Q 2. Write any one of the Euclid's postulate.

Q 3. Write Euclid's axiom 5.

Q 4. By which Euclid's axiom 'If $x + y = 5$, then $x + y - z = 5 - z$'?

Solutions

1. There are seven Euclid's axioms exist.
2. One of the Euclid's postulate is 'All right angles are equal to one another'.
3. Euclid's axiom 5 is 'The whole is greater than the part'.
4.

TR!CK

Using Euclid's axiom 3, 'If equals are subtracted from equals, the remainders are equal'.

If $x + y = 5$, then $x + y - z = 5 - z$

Very Short Answer Type Questions

Q 1. It is known that if $x + y = 10$, then $x + y + z = 10 + z$. Which Euclid's axiom is used?

Q 2. Solve the equation $x - 15 = 20$.

Q 3. Explain when a system of axioms is called consistent.

Q 4. How many lines can be passed through two distinct points?

Q 5. A point P is said to lie between the points M and N. Explain it.

Q 6. Define the condition of a line segment AB, such that point C is called the mid-point of AB.

Q 7. What is a surface?

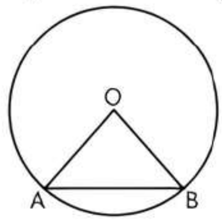
Q 8. Euclid divided the 'elements' into how many books?

Q 9. How many number of propositions are there in 'The Elements'?

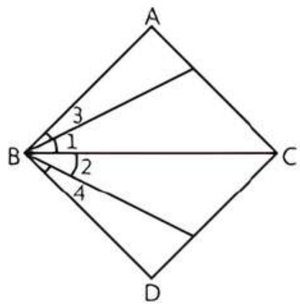
Short Answer Type-I Questions

Q 1. State any two Euclid's axioms.

Q 2. In the given figure, O is the centre of the circle and chord AB is equal to radius of circle. By which Euclid's axiom it can be proved that ΔOAB is an equilateral triangle?



Q 3. In the given figure, we have $\angle 1 = \angle 2$, $\angle 3 = \angle 4$. Show that $\angle ABC = \angle DBC$. State the Euclid's axiom used.



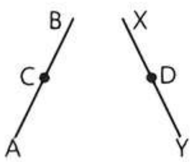
Q 4. Solve the equation $x + 4 = 10$ and state Euclid's axiom used.

Q 5. In a triangle PQR, X and Y are the points on PQ and QR respectively. If PQ = QR and QX = QY, show that PX = RY.

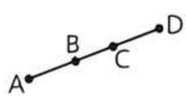
Q 6. If P, Q and R are three points on a line and Q is between P and R, then prove that PR – QR = PQ.

Q 7. Two salesmen make equal sales during the month of August. In September, each salesman doubles his sale of the month of August. Compare their sales in September.

Q 8. In the given figure, AC = XD, C is the mid-point of AB and D is mid-point of XY. Using Euclid's axiom, show that AB = XY.



Q 9. In the given figure, if AC = BD, then prove that AB = CD.



Q 10. If $x + y = 10$ and $x = z$, then show that $z + y = 10$.

Q 11. Consider two 'postulates' given below:

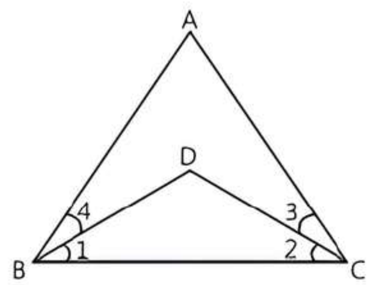
(i) Given any two distinct points A and B, there exists a third point C which is in between A and B.

(ii) There exists at least three points that are not on the same line.

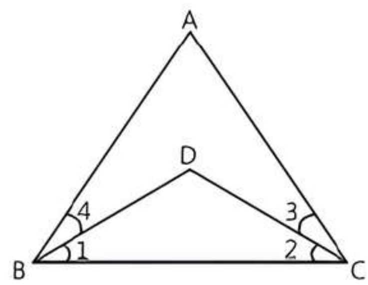
Do these postulates contain any undefined terms? Are these postulates consistent? Do they follow from Euclid's postulates? Explain.

Short Answer Type-II Questions

Q 1. In the given figure, we have $\angle ABC = \angle ACB$, $\angle 3 = \angle 4$. Show that $\angle 1 = \angle 2$.



Q 2. In the given figure, we have $\angle ABC = \angle ACB$ and $\angle 3 = \angle 4$. Show that BD = DC.



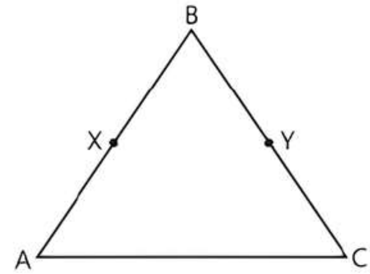
Q 3. Which of the following statements are true and which are false? Give reasons for your answers.

(i) There are an infinite number of lines which pass through two distinct points.

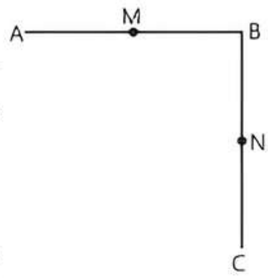
(ii) A terminated line can be produced in definitely on both the sides.

(iii) In the given figure, if AB = PQ and PQ = XY, then AB = XY.

Q 4. In the given figure, we have $BX = \frac{1}{2} AB$, $BY = \frac{1}{2} BC$ and AB = BC. Show that BX = BY.



Q 5. In the given figure,



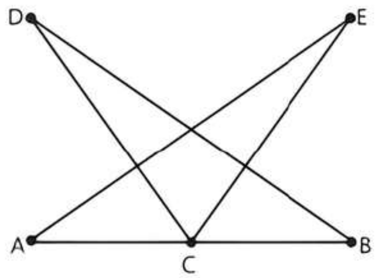
(i) AB = BC, M is the mid-point of AB and N is the mid-point of BC.

Show that AM = NC.

(ii) BM = BN, M is the mid-point of AB and N is the mid-point of BC. Show that AB = BC.

Q 6. PQ is a line segment 12 cm long and R is a point in its interior such that PR = 8 cm. Then find QR, $PQ^2 - PR^2$ and $PR^2 + QR^2 + 2\,PR{\cdot}QR$.

Q 7. In the given figure, AC = BC, ∠DCA = ∠ECB and ∠DBC = ∠EAC. Prove that triangles DBC and EAC are congruent and hence show that DC = EC.



Q 8. Read the following axioms:

(i) Things which are equal to the same things, are equal to one another.

(ii) If equals are added to equals, the wholes are equal.

(iii) Things which are double of the same things, are equal to one another.

Check whether the given system of axioms is consistent or inconsistent.

Q 9. For given four distinct points in a plane, find the number of lines that can be drawn through:

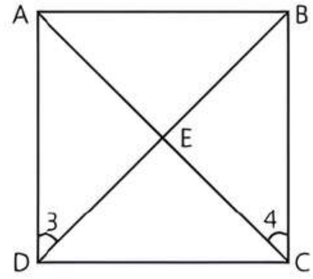
(i) When all four points are collinear.

(ii) When three of the four points are collinear.

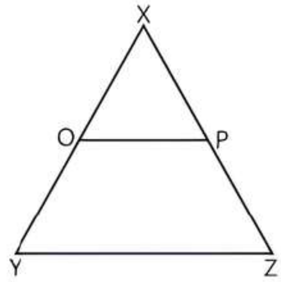
(iii) When no three of the four points are collinear.

Long Answer Type Questions

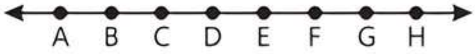
Q 1. A square figure is given ahead. If ∠3 = ∠4, then show that ED = EC.



Q 2. In the figure, if $OX = \frac{1}{2}XY$, $PX = \frac{1}{2}XZ$ and OX = PX, then show that XY = YZ. State which axiom you used here. Also give two more axioms other than the axioms used in the above situation.



Q 3. Using Euclid's axiom, compare lengths AD and AF. State which axiom you used here. Also give two more axioms other than the axiom used in this situation.



Q 4. 'A square is a polygon made up of four line segments, out of which, length of three line segments are equal to the length of fourth one and all its angles are right angles'.

Define the terms used in this definition which you feel are necessary. Are there any undefined terms in this? Can you justify that all angles and sides of a square are equal?

Q 5. Define the following:

(i) Interior point (ii) Bisector of a line

(iii) Between (iv) Mid-point

Solutions

Very Short Answer Type Questions

1. According to Euclid's second axiom, if equals are added to equals, then the wholes are equal.

   Hence, $x + y + z = 10 + z$

2. Given, $x - 15 = 20$

   $\Rightarrow \quad x - 15 + 15 = 20 + 15$

   [Adding 15 on both sides]

TR!CK

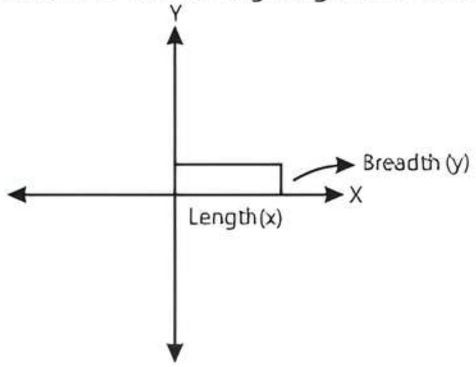
Use Euclid's second axiom, if equals are added to equals, then the wholes are equal.

$\Rightarrow \quad x = 35$

3. A system of axioms is called consistent when it is impossible to deduce from these axioms, a statement that contradicts any axiom or any previously proved statement.

4. Only one line can pass through two distinct points.

5. When three or more points lie on the same line, then they are said to be collinear points. Hence, points M, P and N are collinear points.
6. Mid-point is an interior point of a line which divides the line segment into two equal parts. Hence, for a line segment AB, AC = CB.
7. A surface is one having length and breadth.



8. Euclid divided his famous treatise 'elements' into thirteen chapters each called a book.
9. Euclid deduced 465 propositions in a logical chain using his axioms, postulates, definitions and theorems.

Short Answer Type-I Questions

1. **Euclid's Axioms:**
   (i) Things which are equal to the same thing, are equal to one another.
   (ii) If equals are added to equals, the wholes are equal.

2. $\because$ OA = OB [Radii of same circle]
   OA = AB [Given]
   According to Euclid's first axiom, we have
   AB = OB $\Rightarrow$ OA = OB = AB
   Hence, ΔOAB is an equilateral triangle.

3. Given, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$.

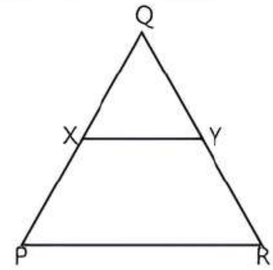
TR!CK
Use Euclid's second axiom, if equals are added to equals, then the wholes are equal.

Now, $\angle 1 + \angle 3 = \angle 2 + \angle 4$
$\Rightarrow$ $\angle ABC = \angle DBC$ **Hence proved**

4. Given equation is $x + 4 = 10$.
   $\Rightarrow$ $x + 4 - 4 = 10 - 4$
   $\Rightarrow$ $x = 6$
   If equals are subtracted from equals, the remainders are equal.
   Hence, Euclid's third axiom is used here.

5. Given, PQ = QR and QX = QY

TR!CK
Use Euclid's third axiom, if equals are subtracted from equals, the remainders are also equal.



$\therefore$ PQ – QX = QR – QY
$\Rightarrow$ PX = RY **Hence proved**

6. In the figure given below PQ coincides with PR – QR. So, according to Euclid's fourth axiom, things which coincide with one another are equal to one another.

$\therefore$ PR – QR = PQ **Hence proved**

7. Let x and y be the sales of two salesmen in the month of August.
   $\therefore$ $x = y$ [Given]
   In the month of September, each salesman doubles his sale of the month of August.
   $\Rightarrow$ $2x = 2y$

TR!CK
Use Euclid's sixth axiom, things which are double of the same things, are equal to one another.

Hence, the sales of two salesmen are equal in the month of September.

8. In question figure, AB = 2AC
   [$\because$ C is the mid-point of AB]
   XY = 2XD
   [$\because$ D is the mid-point of XY]
   Also, AC = XD [Given]
   $\Rightarrow$ 2AC = 2XD
   According to Euclid's sixth axiom, things which are double of the same things, are equal to one another.
   $\therefore$ AB = XY **Hence proved**

9. Given, AC = BD
   Now, AC = AB + BC
   and BD = BC + CD
   $\because$ AC = BD [Given]
   $\therefore$ AB + BC = BC + CD

TR!CK
Use Euclid's third axiom, if equals are subtracted from equals, the remainders are equal.

$\Rightarrow$ AB + BC – BC = BC + CD – BC
$\therefore$ AB = CD **Hence proved**

10. Given, $x + y = 10$ ...(1)

and $x = z$ ...(2)

TR!CK

Use Euclid's third axiom, if equals are subtracted from equals, the remainders are also equal.

On subtracting y from both sides of eq. (1), we get

$$x + y - y = 10 - y$$

$\Rightarrow$ $x = 10 - y$

or $z = 10 - y$ [From eq. (2)] ...(3)

Now, according to Euclid's second axiom, if equals are added to equals, the wholes are equal.

On adding y to both sides of eq. (3), we get

$$z + y = 10 - y + y$$

$\Rightarrow$ $z + y = 10$ **Hence proved**

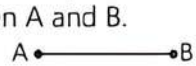
11. In postulate (i), between A and B, there remains an undefined term which appeals to our geometric intuition.

These postulates are consistent. They do not contradict each other. Both of these postulates do not follow Euclid's postulates. However, they follow the axiom given below.

Given two distinct points so, there is a unique line that passes through them.

(i) Let AB be a straight line.

There are an infinite number of points composing this line. Choose any except the two end points A and B. This point lies between A and B.



(ii) If there are only two points, they can always be connected by a straight line (From Euclid's postulate). Therefore, there have to be at least three points for one of them not to fall on the straight line between the other two.

Short Answer Type-II Questions

1. Given, $\angle 3 = \angle 4$ or $\angle 4 = \angle 3$ ...(1)

and $\angle ABC = \angle ACB$

$\therefore$ $\angle 1 + \angle 4 = \angle 2 + \angle 3$ ...(2)

TR!CK

Use Euclid's third axiom, if equals are subtracted from equals, then remainders are equal.

On subtracting eq. (1) from eq. (2), we get

$$\angle 1 + \angle 4 - \angle 4 = \angle 2 + \angle 3 - \angle 3$$

$\therefore$ $\angle 1 = \angle 2$ **Hence proved**

2. Given, $\angle ABC = \angle ACB$ ...(1)

and $\angle 4 = \angle 3$ ...(2)

TR!CK

Use Euclid's third axiom, if equals are subtracted from equals, then remainders are also equal.

On subtracting eq. (2) from eq. (1), we get

$$\angle ABC - \angle 4 = \angle ACB - \angle 3$$

$\Rightarrow$ $\angle 1 = \angle 2$

Now, in Δ BDC,

$$\angle 2 = \angle 1$$

Sides opposite to equal angles are equal.

$\therefore$ BD = DC **Hence proved**

3. (i) False;

Through two distinct points, only one line can pass.

(ii) True;

A terminated line or line segment can be produced indefinitely on both sides to give a line.

(iii) True;

From the axiom that if two things are separately equal to a third thing, then they are equal to each other.

Hence, if AB = PQ

and PQ = XY then AB = XY

4. Given, $BX = \frac{1}{2}AB$, $BY = \frac{1}{2}BC$ ...(1)

and AB = BC

$\Rightarrow$ $\frac{1}{2}AB = \frac{1}{2}BC$ [Multiply both sides by $\frac{1}{2}$]

...(2)

TR!CK

Use Euclid's seventh axiom, things which are halves of the same things, are equal to one another.

From eqs. (1) and (2), we get

BX = BY **Hence proved**

5. (i) Given, AB = BC ...(1)

and M is the mid-point of AB.

$\Rightarrow$ $AM = MB = \frac{1}{2}AB$ ...(2)

and N is the mid-point of BC.

$\Rightarrow$ $BN = NC = \frac{1}{2}BC$ ...(3)

From eq. (1), AB = BC

On multiplying both sides by $\frac{1}{2}$, we get

$$\frac{1}{2}AB = \frac{1}{2}BC$$

TR!CK

Use Euclid's seventh axiom, things which are halves of the same things, are equal to one another.

$\Rightarrow$ AM = NC [From eqs. (2) and (3)]

Hence proved

(ii) Given, BM = BN ...(1)

and M is the mid-point of AB.

$\Rightarrow$ $AM = BM = \frac{1}{2}AB$

$\Rightarrow$ $2AM = 2BM = AB$ ...(2)

and N is the mid-point of BC.

$\Rightarrow \quad BN = NC = \frac{1}{2} BC$

$\Rightarrow \quad 2BN = 2NC = BC$ ...(3)

TR!CK

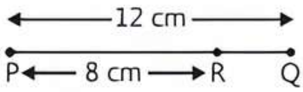
Use Euclid's sixth axiom, things which are double of the same things, are equal to one another.

$\therefore \quad 2BM = 2BN$

$\Rightarrow \quad AB = BC$ [From eqs. (2) and (3)]

Hence proved

6. Given, PQ = 12 cm and PR = 8 cm



Here, point R lies in the interior of PQ.

$\therefore \quad PR + QR = PQ$

$\Rightarrow \quad 8 + QR = 12$ ...(1)

TR!CK

Use Euclid's third axiom, if equals are subtracted from equals, the remainders are equal.

On subtracting 8 from both sides of eq. (1), we get

$8 + QR - 8 = 12 - 8$

$\Rightarrow \quad QR = 4$ cm

Now, $PQ^2 - PR^2 = (12)^2 - (8)^2$

$= 144 - 64 = 80 \text{ cm}^2$

Also, $PR^2 + QR^2 + 2PR \cdot QR$

$= (PR + QR)^2$

$= (8 + 4)^2 = (12)^2 = 144 \text{ cm}^2$

Hence, the values of QR = 4 cm, $PQ^2 - PR^2 = 80 \text{ cm}^2$ and $PR^2 + QR^2 + 2PR \cdot QR = 144 \text{ cm}^2$.

7. Given, $\angle DCA = \angle ECB$ and $\angle DBC = \angle EAC$

TR!CK

Use Euclid's second axiom, if equals are added to equals, then the wholes are equal.

$\Rightarrow \quad \angle DCA + \angle DCE = \angle ECB + \angle DCE$

$\Rightarrow \quad \angle ECA = \angle DCB$ ...(1)

Now, in triangles ΔDBC and ΔEAC,

$\angle DCB = \angle ECA$ [From eq. (1)]

$BC = AC$ [Given]

and $\angle DBC = \angle EAC$ [Given]

$\therefore \quad \Delta DBC \cong \Delta EAC$ [By ASA criterion]

$\Rightarrow \quad DC = EC$ [By CPCT]

Hence proved

8. To check given system is consistent or inconsistent, we have to find that whether we can deduce a statement from these axioms which contradicts any axiom or not.

Some of Euclid's axioms are:

(i) Things which are equal to the same things, are equal to one another.

(ii) If equals are added to equals, the wholes are equal.

(iii) Things which are double of the same things, are equal to one another.

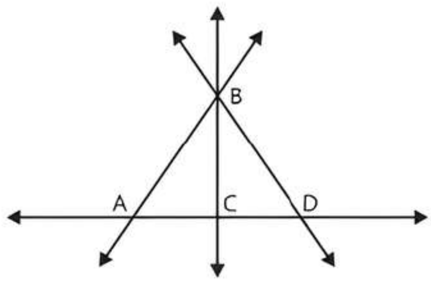
So, we cannot deduce any statement from these axioms which contradicts any axiom. Hence, given system of axioms is consistent.

9. (i) Consider the points given as A, B, C and D.

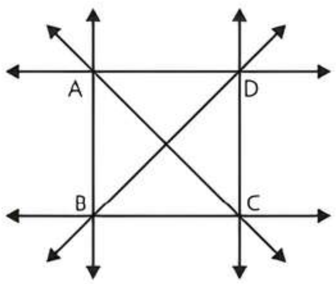
When all the four points are collinear, we have one line $\overleftrightarrow{AD}$.

(ii) When three of the four points are collinear, we have four lines as

$\overleftrightarrow{AB}, \overleftrightarrow{BC}, \overleftrightarrow{BD}$ and $\overleftrightarrow{AD}$



(iii) When no three of the four points are collinear, we have six lines as $\overleftrightarrow{AB}, \overleftrightarrow{BC}, \overleftrightarrow{AC}, \overleftrightarrow{AD}, \overleftrightarrow{BD}$ and $\overleftrightarrow{CD}$.



Long Answer Type Questions

1. We know that each adjacent angle of a square is 90°.

$\therefore \quad \angle ADC = \angle BCD$ [Each 90°] ...(1)

Also, $\angle 3 = \angle 4$ [Given] ...(2)

According to Euclid's third axiom, if equals are subtracted from equals, the remainders are equal.

On, subtracting eq. (2) from eq. (1), we get

$\angle ADC - \angle 3 = \angle BCD - \angle 3$

$\Rightarrow \quad \angle ADC - \angle 3 = \angle BCD - \angle 4$ [From eq. (2)]

$\Rightarrow \quad \angle EDC = \angle ECD$

$\Rightarrow \quad ED = EC$ [Sides opposite to equal angles are equal] **Hence proved**

2. Given, OX = PX

From figure, $OX = \frac{1}{2}XY$ and $PX = \frac{1}{2}XZ$

$\therefore \quad \frac{1}{2}XY = \frac{1}{2}XZ$

TR!CK

Use Euclid's seventh axiom, things which are halves of the same things, are equal to one another.

$\Rightarrow \quad XY = XZ$

Two Other Axioms:

1. Things coincide with one another are equal to one another. *e.g.*, If AB coincide with XY such that A falls on X and B falls on Y, then AB = XY.
2. The whole is greater than the part. *e.g.*, If $z = x + y$, where $x, y \neq 0$, then $z > x$ and $z > y$.

3. From figure, AD is part of AF.

$\therefore \quad AD < AF$

According to Euclid's fifth axiom, the whole is greater than the part.

Two More Axioms:

1. If equals are added to equals, the wholes are equal. *e.g.*, If $m\angle 1 = m\angle 2$ then, $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$
2. If equals are subtracted from equals, the remainders are equal. *e.g.* If $m\angle 1 = m\angle 2$, then $m\angle 1 - m\angle 3 = m\angle 2 - m\angle 3$.

4. The terms that are needed to be defined are:
   1. **Polygon:** A simple closed figure made up of three or more line segments.
   2. **Line Segment:** Part of a line with two end points.
   3. **Angle:** A figure formed by two rays with common initial point.
   4. **Right Angle:** Angle whose measure is 90°.

TR!CK

Use Euclid's fourth postulate, all right angles are equal to one another.

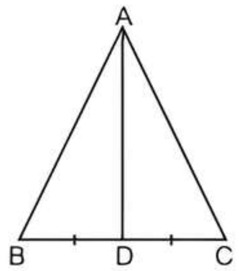
In a square, all angles are right angles, therefore all angles are equal.

Three line segments are equal to fourth line segment. [Given]

Therefore, all the four sides of a square are equal. Also, according to Euclid's first axiom, things which are equal to the same things, are equal to one another.

5. **(i) Interior Point:** A point P is called interior point of line segment AB, if $P \in AB$ but P neither lies on A nor on B, *i.e.*, $P \neq A$ and $P \neq B$.

(ii) Bisector of a Line: A line passing through mid-point of the line is called bisector of the line. From figure,



BD = DC

So, AD is the bisector of BC.

(iii) Between: Point C is said to lie between the points A and B, if:

(a) A, B and C are collinear.

(b) AC + CB = AB

(iv) Mid-point: The point M is called mid-point of the line segment AB, if M lies on AB such that AM + MB = AB and AM = MB.

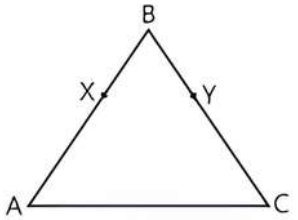
Chapter Test

Multiple Choice Questions

Q 1. 'Lines are parallel if they do not intersect' is stated in the form of:

a. a proof
b. a definition
c. a postulate
d. an axiom

Q 2. In the adjoining figure, if AB = BC and BX = BY, then:



a. AX = CY
b. AC = XY
c. AY = CX
d. None of these

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4) *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
c. Assertion (A) is true but Reason (R) is false.
d. Assertion (A) is false but Reason (R) is true.

Q 3. Assertion (A): For two distinct points, there is a unique line that passes through them.

Reason (R): If A, B and C are three points on a line and B lies between A and C, then AB + BC = AC.

Q 4. Assertion (A): The whole is always greater than the part.
Reason (R): A cake when it is whole or complete, assume that it weighs 2 pounds but when a part is taken out from it and measured, its weight will be smaller than the previous measurement.

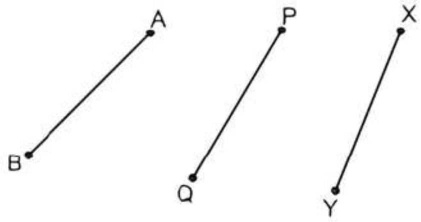
Fill in the Blanks

Q 5. .... are the assumptions which are obvious universal truth. (axioms/Theorem)

Q 6. Things which are double of the same things, are .... to one another.

True/False

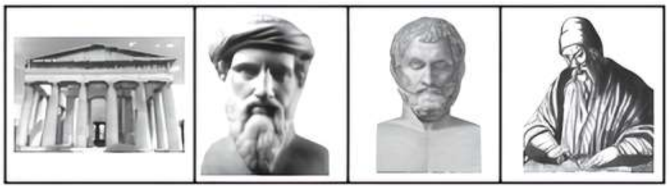
Q 7. In figure, if AB = PQ and PQ = XY, then AB = XY.



Q 8. There are an infinite number of lines which pass through two distinct points

Case Study Based Question

Q 9. A national Public School organised an education trip to a museum. Almost all the students of class IX went to the trip with their teacher of Mathematics. They saw many pictures of mathematician and read about their contributions in the field of Mathematics. After visiting the museum, teacher asked the following questions from the students.



On the basis of the above information, solve the following questions:

(i) Who was a student of Pythagoras?

(ii) Name of mathematician who is visible in the last picture?

OR

In which country thales belongs to?

(iii) Is it true that theorem needs a proof?

Very Short Answer Type Questions

Q 10. Solve the equation $x - 10 = 15$.

Q 11. If three points lies in a line, then how many lines can be passed through three distinct points?

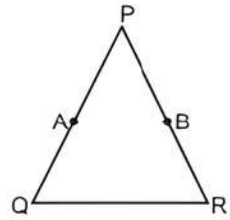
Short Answer Type-I Questions

Q 12. State any two Euclid's axioms.

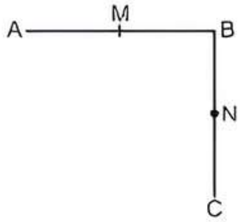
Q 13. Solve the equation $y + 6 = 12$ and state that Euclid's.

Short Answer Type-II Questions

Q 14. In the given figure, we have $AP = \frac{1}{2} PQ$, $PB = \frac{1}{2} PR$ and $PQ = PR$. Show that $AP = PB$.



Q 15. In the given figure,



(i) AB = BC, M is the mid-point of AB and N is the mid-point of BC. Show that AM = NC.

(ii) BM = BN, M is the mid-point of AB and N is the mid-point of BC. Show that AB = BC.

Long Answer Type Question

Q 16. A square figure is given below. If $\angle 3 = \angle 4$, then show that ED = EC.